



MATHEMATICS

MAXIMUM MARKS: 80

TIME ALLOWED: THREE HOURS

(CANDIDATES ARE ALLOWED ADDITIONAL 15 MINUTES FOR ONLY READING THE PAPER. THEY MUST NOT START WRITING DURING THIS TIME.)

- *This Question Paper consists of three sections A, B and C.*
- *Candidates are required to attempt all questions from Section A and all questions EITHER from Section B OR Section C.*
- *Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.*
- *Section B: Internal choice has been provided in one question of two marks and one question of four marks.*
- *Section C: Internal choice has been provided in one question of two marks and one question of four marks.*
- *All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.*
- *The intended marks for questions or parts of questions are given in brackets [].*
- *Mathematical tables and graph papers are provided.*

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SECTION A - 65 MARKS

Question - 1

(i) If a matrix is both symmetric and skew-symmetric, then A is a

- (a) Scalar matrix
- (b) Diagonal matrix
- (c) Zero matrix
- (d) Square matrix

(ii) If $\int \frac{\sqrt{2} \sin x}{\sin(x - \frac{\pi}{4})} dx = ax + b \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + C$, find the value of a and b.

- (a) $a=1, b=1$
- (b) $a=2, b=3$



(c) $a=0, b=5$

(d) $a=9, b=4$

(iii) Evaluate : $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$.

(a) 1

(b) 2

(c) 3

(d) 4

(iv) The integrating factor of differential equation $(1-x^2)\frac{dy}{dx} - xy = 1$ is

(a) $-x$

(b) $\frac{x}{1+x^2}$

(c) $\sqrt{1-x^2}$

(d) $\frac{1}{2}\log(1-x^2)$

(v) Yash is appearing for an exam of Maths and Physics

let event A = Yash passes in Maths

let event B = Yash passes in Physics

given $P(A) = 0.30$ and $P(B) = 0.40$ and $P(A \text{ and } B) = 0.20$

Which one of the following is true about the events A and B?

(a) They are mutually exclusive but not independent

(b) They are independent but not mutually exclusive

(c) They are both mutually exclusive and independent

(d) They are neither mutually exclusive nor independent

(vi) If A and B are square matrices of same order such that $|A| = -2$ and $B = I$, then find the value of $|B|$.

(a) $\frac{3}{2}$

(b) $-\frac{1}{2}$

(c) $\frac{4}{5}$

(d) $-\frac{7}{2}$

(vii) Three graphs are given below: From the given options, choose the correct option.

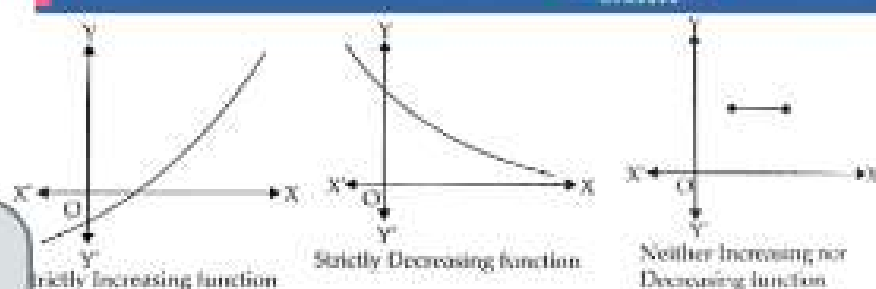


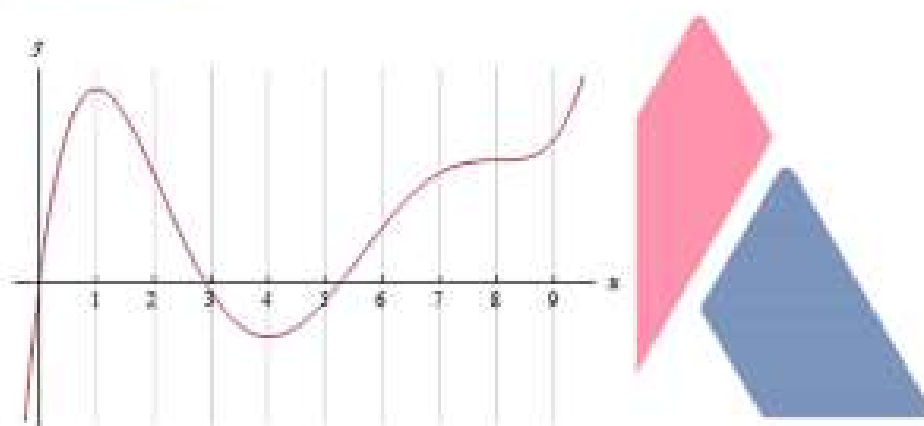
Figure (i) and (ii) are correct

(b) Figure (ii) and (iii) are correct

(c) Figure (iii) and (i) are correct

(d) All the figures are correct

(viii) Determine the intervals on which the following function increases.



(a) $(-\infty, -7)$ & $(5, \infty)$

(b) $(-\infty, -7)$ & $(-5, \infty)$

(c) $(-\infty, -7)$ & $(5, -\infty)$

(d) $(\infty, -7)$ & $(5, \infty)$

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(ix) Let $f: N \rightarrow N$ be a function defined by $f(x) = 2x$. Then, f is

(a) Both one-one and onto

(b) One-one but not onto

(c) Onto but not one-one

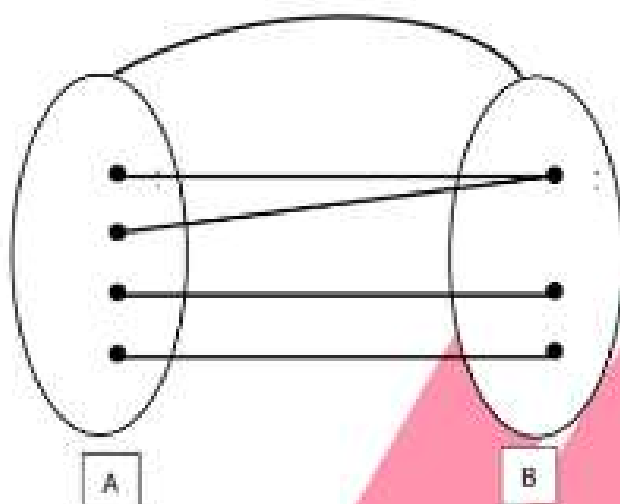
(d) Neither one-one nor onto

(x) Assertion (A) : $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix.

Reason (R) : A square matrix is called a scalar matrix if all the non-diagonal elements are zero and the diagonal elements are equal.

- (a) Both A and R are correct and R is the correct explanation of A.
 (b) Both A and R are correct but R is not the correct explanation of A.
 (c) A is correct but R is not correct.
 (d) A is not correct but R is correct.

(xi) Identify if the given relation is one-one or not. Give reason.



(xii) Write the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$.

(xiii) The relation R on N, defined by $R = \{(x, y) : y = x + 5, x < 4\}$, is

- (a) Not transitive
 (b) Reflexive but not symmetric
 (c) Not reflexive not symmetric but transitive
 (d) Symmetric but not reflexive
- (xiv) A problem of statistics is given to two students whose chances of solving it are $\frac{1}{3}$ and $\frac{1}{5}$. What is the probability that the problem is solved?
- (xv) If $(A \cap B) = \frac{1}{3}$, $P(A \cup B) = \frac{5}{6}$ and $P(A) = \frac{1}{2}$, then which one of the following is correct?
- (a) A and B are independent events
 (b) A and B are mutually exclusive events
 (c) A and B are equally likely events
 (d) None of these

Question -2

- (i) The table below shows some of the values of differentiable functions u and v and their derivatives.

x	$u(x)$	$u'(x)$	$v(x)$	$v'(x)$
1	0	2	-1	5
2	2	5	3	2
3	2	-1	3	0

If $r(x) = v(u(x))$ find $r'(2)$.

- (ii) If $e^{x+y} = xy$, then prove that $\frac{dy}{dx} = \frac{y(1-x)}{x(1-y)}$

Question -3

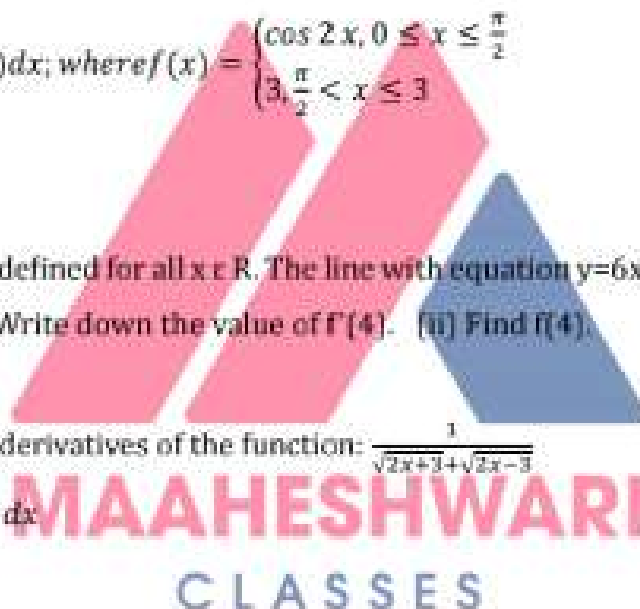
Evaluate: $\int_0^3 f(x) dx$; where $f(x) = \begin{cases} \cos 2x, & 0 \leq x \leq \frac{\pi}{2} \\ 3, & \frac{\pi}{2} < x \leq 3 \end{cases}$

Question -4

The function f is defined for all $x \in \mathbb{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$. (i) Write down the value of $f'(4)$. (ii) Find $f(4)$.

Question -5

- (i) Find all the anti-derivatives of the function: $\frac{1}{\sqrt{2x+3} + \sqrt{2x-3}}$
- (ii) Evaluate: $\int x e^{3x} dx$



Question -6

Find the domain and the range of the function f defined by $f(x) = \frac{x+2}{|x+2|}$

Question -7

If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, prove that $\sin y = \tan^2 \frac{x}{2}$

Question -8

Evaluate: $\int \frac{3x+1}{2x^2-2x+3} dx$

Question -9

- (i) For what choice of a and b , is the function $f(x) = \begin{cases} x^2, & x \leq c \\ ax + b, & x > c \end{cases}$ differentiable at $x = c$?
- (ii) If $y = (x + \sqrt{1 + x^2})^n$, then prove that $(1 + x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = n^2 y$.

Question -10

Four friends toss a coin each. They pay if exactly one gets heads (or exactly one gets tails — but here they decide: if exactly one result is different from the other three, that person pays). If all four results are same or if two heads two tails, they toss again.

- (a) What is the probability that in one round, exactly one person gets heads?
 (b) What is the probability they need to toss again?
 (c) What is the probability that they decide in the third round?

OR

A disease has three risk groups in a population:

- High risk: 10% of population, disease prevalence 20%
- Medium risk: 30% of population, disease prevalence 5%
- Low risk: 60% of population, disease prevalence 1%

A person tests positive for the disease.

- (a) Represent the events.
 (b) Find the probability the person is from the High-risk group given they have the disease.
 (c) Which risk group is most likely given disease status?

Question -11

A shopkeeper has 3 varieties of pen 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of 21. Jeevan purchased 4 pens of 'A' variety, 3 pen say 'B' variety and 2 pens of 'C' variety for 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹70.

Form a set of simultaneous equations for the above information.

Solve the set of equations formed in (i) by matrix method.

- (iii) Hence, find the cost of each variety of pen.

Question -12

- (i) Find the particular solution of the differential equation $(y - \sin x)dx + (\tan x)dy = 0$ satisfying the condition that $y = 0$ when $x=0$.
- (ii) Evaluate : $\int \frac{dx}{x^4 + 16}$

Question -13

- (i) An open box with a square base is to be made out of a given quantity of cardboard of area c^2 . Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$.
- (ii) Of all the closed right circular cylindrical cans of volume $128\pi\text{cm}^3$, find the dimensions of the can which minimum surface area.

Question -14

The probability distribution of a random variable X is given as under:

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant Calculate:

- (i) the value of k (ii) $P(X \geq 4)$ (iii) $E(X)$

Section B 15 MARKS

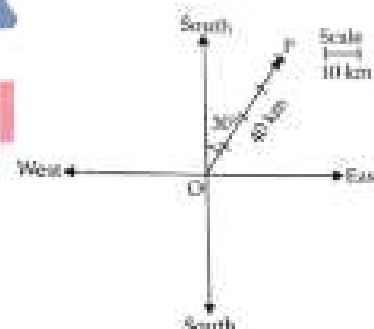
Question -15

In sub-parts(i) and (ii) choose the correct options and in sub-parts (iii) to (v) , answer the questions as instructed.

- (i) Observe the vector diagram shown below and the expression that follow:

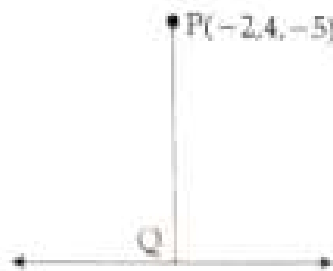
Here, vector OP represents:

- (a) displacement of 40km, 30° east of north
 (b) displacement of 30km, 40° east of north
 (c) displacement of 40km, 30° north of east
 (d) displacement of 30km, 40° north of east



- (ii) Find a vector of magnitude of 10 units and parallel to the vector $2\hat{i} + 3\hat{j} - \hat{k}$.
- (iii) Find the position vector of a point R which divides the line joining the two-points P and Q with position vectors $2\hat{i} + \hat{j}$ and $\hat{i} - 2\hat{j}$ respectively in the ratio of 2 : 1 externally.
- (iv) The planes $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are
- (a) parallel (b) intersect on Y axis
 (c) perpendicular (d) pass through $(0, 0, \frac{5}{4})$

- (v) Find the distance of the point $(-2, 4, 5)$ from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.



Question -16

- (i) Find the area of the triangle whose adjacent sides are $\hat{i} + 4\hat{j} - \hat{k}$ and $\hat{i} + \hat{j} + 2\hat{k}$.
 (ii) For any two non-zero vectors $\vec{a}, \vec{b}$, if $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then find the angle between them.

Question -17

- (i) Show that $\frac{4-x}{-1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect each other. Also find out the point of intersection.
 (ii) Find the equation of the plane passing through the points $(0, 4, -3)$ and $(6, -4, 3)$ if the sum of their intercepts on three axes is 0.

Question -18

If the area is bounded the parabola $y^2 = 16x$ and the line $y = 4mx$ is $\frac{1}{12}$ sq units, then using integration, find the value of m . ($m > 0$)

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Question -19

In sub-parts (i) and (ii) choose the correct options and in sub-parts (iii) to (v), answer the questions as instructed.

- (i) The average cost function associated with producing and marketing x units of an item is given by $AC = 3x - 11 + \frac{10}{x}$. Then MC at $x = 2$ is
 (a) 2 (b) 5 (c) 3 (d) 1
 (ii) If $\sigma_x = 5$, $r = -\frac{1}{2}$ and $b_{yx} = -\frac{2}{7}$, then the value of σ_y is
 (a) $\frac{7}{20}$ (b) $\frac{20}{7}$ (c) $-\frac{20}{7}$ (d) $-\frac{7}{20}$
 (iii) The straight line graph of the linear equation $y = a + bx$, slope will be upward if;
 (a) $a = 0$ (b) $b > 0$ (c) $b < 0$ (d) $b \neq 0$

- (iv) If the marginal revenue function for a commodity is $MR = 9 - 4x^2$, find the demand function.
- (v) Find the marginal cost function (MC), if the cost function is : $C(x) = \frac{x^3}{3} + 5x^2 - 16x + 2$

Question -20

- (i) The average cost function associated with producing and marketing x units of an item is given by, $AC = 2x - 11 + \frac{50}{x}$ Find: (i) the total cost function and marginal cost function (ii) the range of values of the output x , for which AC is increasing.

OR

- (ii) The manufacturing cost of an item is Rs. 3 per item, fixed cost is Rs. 900 and the labour cost is Rs. $\frac{x^2}{100}$ for x items produced. How many items must be produced to have average cost minimum.

Question -21

For observation of pairs (x, y) of the variables X and Y , the following results are obtained.

$\sum x = 125, \sum y = 100, \sum x^2 = 1650, \sum y^2 = 1500, \sum xy = 50, n = 25$. find the equation of the line of regression of x on y . Estimate the value of x , if $y = 5$.

Question -22

- (i) Find the line of regression line y on x for the data:

X	1	2	3	4	5
Y	7	6	5	4	3

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OR

- (ii) Two tailors P and Q earn Rs. 150 and Rs. 200 per day respectively. P can stitch 6 shirts and 4 trousers a day, while Q can stitch 10 shirts and 4 trousers per day. How many days should each work to produce at least 60 shirts and 32 trousers at minimum labour cost?